Unit 3 Guided Notes

Polynomial Functions

Standards: **A.APR.4, A.APR.6, A.REI.11, A.SSE.2, F.BF.1b, F.BF.3, F.IF.4, F.IF.5, F.IF.6, F.IF.7c, F.IF.8, F.IF.9**

Clio High School – Algebra 2A

Name: ________________________________ Period: ______________

Need help? Support is available!

- Miss Seitz's tutoring: See schedule in classroom
- Website with all videos and resources

www.msseitz.weebly.com

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Concept #	What we will be learning...	Text
#1	**Introduction to Polynomials** ☐ Tell whether an expression is a polynomial or not a polynomial ☐ Identify end behavior from standard form of a polynomial ☐ List and explain the important parts of a graph or table such as; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity ☐ Sketch a graph given a verbal description.	5.1
#2	**Roots of Polynomials** ☐ Factor out a GCF ☐ Identify roots of a polynomial from factored form ☐ Make connection between roots, zeros, solutions, and x-intercepts ☐ Make a rough sketch of the graph of a polynomial given roots and standard form ☐ Change the form of an expression to identify key features such as slope, intercepts and end behavior	5.2
#3	**Transformation of Polynomials** ☐ Explain how k changes the graph depending on where k is and what number k is. ☐ Find the value of k given graphs ☐ Compare properties of two functions represented in a different way	5.9
#4	**Operations on Polynomials** ☐ I can simplify polynomial expressions ☐ I can multiply polynomials ☐ Put functions together using addition, subtraction, multiplication, and division	6.6
#5	**Solving Polynomials** ☐ Explain why the x-coordinates of the points where the graphs of two functions meet are solutions	5.3
#6	**Dividing Polynomials** ☐ Use the box method to divide a polynomial by (x-a). ☐ Write the answer in the correct form with any remainder written over the (x-a). ☐ Determine if (x-a) is a factor of a polynomial ☐ Divide a polynomial by a polynomial of the same or lesser degree ☐ Write the answer in the correct form with any remainder written over the bottom polynomial	5.4
#7	**Using Polynomial Identities** ☐ Show that the two sides of polynomial "formula" are equal ☐ Use the three polynomial "formulas" to explain numerical relationships	CB 5.5
#8	**Polynomial Models in the Real World** ☐ Explain properties of a real world situation from complex equations ☐ Relate the domain of a function to its graph ☐ Identify the input values that make sense in a real-world situation ☐ Calculate the average rate of change of a function from an equation and/or table ☐ Interpret and estimate the average rate of change of a function given a graph	5.8

#1	**Introduction to Polynomials** **Text: 5.1** ☐ Tell whether an expression is a polynomial or not a polynomial ☐ Identify end behavior from standard form of a polynomial ☐ List and explain the important parts of a graph or table such as; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity ☐ Sketch a graph given a verbal description. Vocabulary: monomial, degree of a monomial, polynomial, degree of a polynomial, end behavior, algebraic expression, leading coefficient, relative maximum, relative minimum, standard form of a polynomial, symmetries

Degree of Monomials & Polynomials	
A M________________ is a real number, variable, or a product of a real number and one or more variables with whole number exponents. **Example:** 8, 5x, $3x^2$	The D_____________ of a MONOMIAL is the value of the exponent in the monomial. **Example:** $3x^5$ is degree 5.A
P__________________ is a real number, variable, or a product of a real number and one or more variables with whole number exponents. **Example:** 5x, 6x + 3, $7x^2 + 2x$	The D____________ of a POLYNOMIAL is the greatest degree among the monomial terms of the polynomials. ***(This value tells the maximum number of Z________)***. **Example:** $7x^2 + 2x$ is degree 2

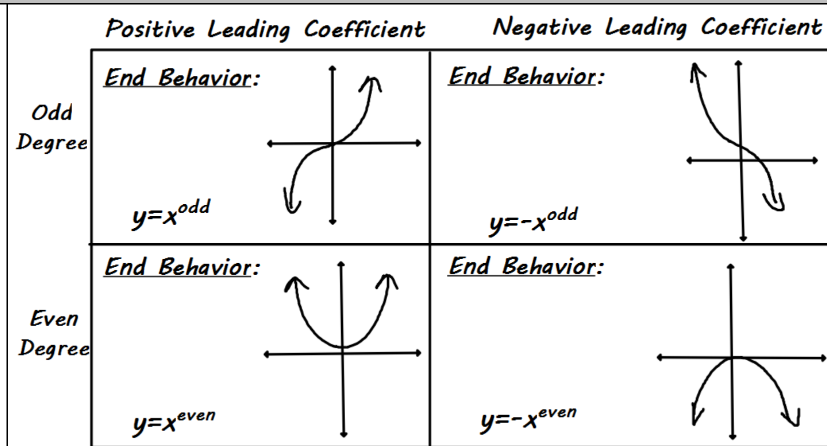
STANDARD FORM of a Polynomial	
The S________________ F___________ of a polynomials function arranges the terms by the D______________ in ***DESCENDING*** numerical order.	**<u>Example 1:</u>** Write in standard form. $5x + 2 + x^2 - 6x^5$

You Try It! Write the polynomial in standard form and state the degree of the polynomials

1.) $3x^3 + 2x^4 - 2x + 4x^2 + 1$

2.) $4n^5$

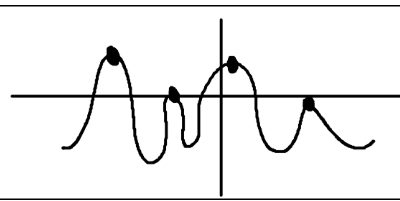
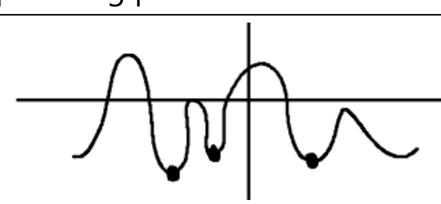
Definitions	
The L____________ C______________ is the coefficient of the variable with the highest degree. **Example:** $5x^3 + 2x$ Leading Coefficient: 5	The C_____________ T_________ is the number without the variable. **Example:** $-3x^7 + 1$ Constant term: 1
The T_____________ P___________ are the places where the graph changes direction.	The E______ B________________ describes the direction of the graph as you move from left to right away from the origin

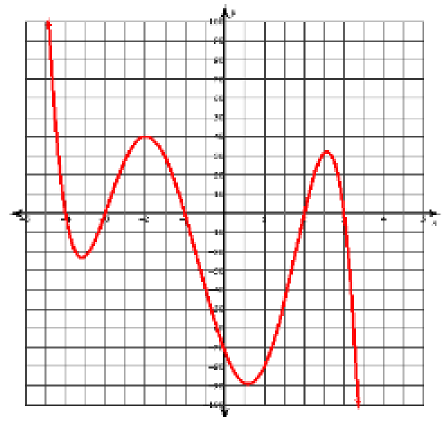
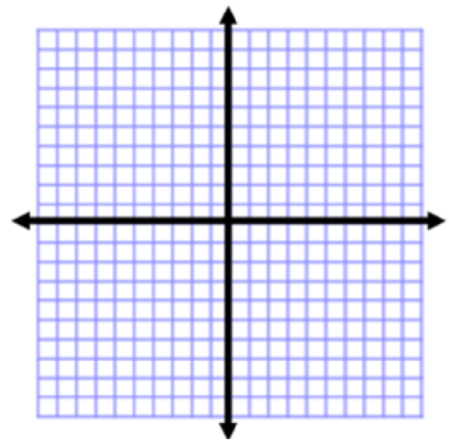
End Behavior	
End Behavior is due to the following: ✓ Degree ✓ Sign of leading coefficient	
<u>Example 2:</u> What is the end behavior of the graph? $y = 4x^3 - 3x$	**<u>Example 3:</u>** Describe the end behavior of $y = -4x^3 + 2x + 7$

You Try It! Describe the end behavior of each

3.) $y = -7x^5 + 2x$

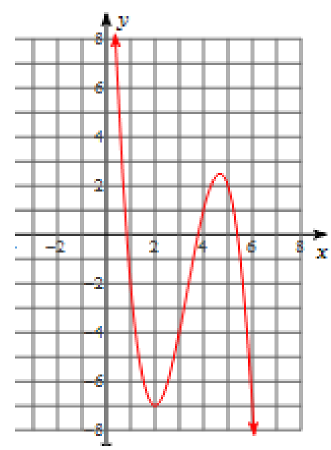
4.) $y = 2x^2 + 3x + 1$

Relative Maximums and Minimums	
A Relative M_____________ is the value of a function on an up to down turning point.	A Relative M_____________ is the value of a function on a down to up turning point.
	

<u>Example 4:</u> Look at the graph and describe 1. Where the graph increases or decreases 2. End Behaviors 3. Zeros of the function 4. Maximums and Minimums	
<u>Example 5:</u> Create a sketch using the verbal description Degree: 4 Leading Coefficient: 2 Y-intercept: (0, 3) Max # of real zeros: 4 End Behavior: up/up	

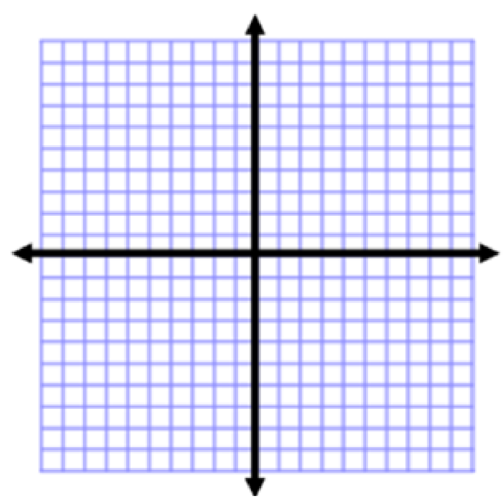
You Try It!

5.) Look at the graph and describe where the graph increases and decreases, and what the end behaviors, zeros, maximums and minimums are.



6.) Create a sketch

Degree: 3
Leading Coefficient: -1
y-intercept (0,2)
Max. # real zeros: 3
End Behavior: UP/DN



#2	**Roots of Polynomials** **Text: 5.2** ☐ Factor out a GCF ☐ Identify roots of a polynomial from factored form ☐ Make connection between roots, zeros, solutions, and x-intercepts ☐ Make a rough sketch of the graph of a polynomial given roots and standard form ☐ Change the form of an expression to identify key features such as slope, intercepts and end behavior Vocabulary: factor, factored form, greatest common factor (GCF), roots, solutions, x-intercepts, zeros

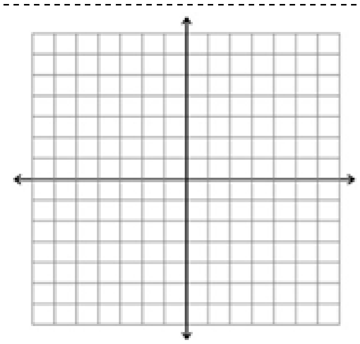
Definitions	
The common factor of each term of an expression that has the greatest coefficient and the greatest exponent is called the G_______________C______________F______________ . *Think biggest number and variable that all terms have in common*	
FACTORED FORM is a polynomial written in the following form: GCF ( )( )	
****For help with factoring, see notes for Unit 2 Concept 3****	
Example 1: Factor out the GCF of $3x^3 + 6x^2 + 3x$	**Example 2:** Put $x^3 - 2x^2 - 15x$ in factored form

Solving Polynomial Equations by Factoring	
R________, Z________, S_____________, and X-______________ are any values for x that make f(x) = 0.	
Example 3: Solve $x^3 - 2x^2 - 15x$	
Factor out GCF	
Factor what's left	
Set each factor=0	

You Try It! Solve by factoring

1.) $y = x^3 + 3x^2 - 10x$

2.) $y = x^4 - 8x^3 + 16x^2$

Graphing a Rough Sketch	
<u>Example 4:</u> $y = (x + 2)(x - 1)(x - 3)$	
Find the zeros (set each factor = 0)	
Pick points between each zero & plug into equation to find y-values	
Identify the end behavior	
Connect points and graph	

You Try It! Find the zeros of the function, then make a rough sketch.

3.) $y = x(x - 2)(x + 5)$

#3	**Transformations of Polynomials** **Text: 5.9** ☐ Explain how k changes the graph depending on where k is and what number k is. ☐ Find the value of k given graphs ☐ Compare properties of two functions represented in a different way Vocabulary: even function, odd function, vertical shift, horizontal shift, vertical dilation, horizontal dilation, reflection

<table>
<tr><th colspan="2">Definitions</th></tr>
<tr><td>Horizontal Shift f(x + k)</td><td>Vertical Shift f(x) + k</td></tr>
<tr><td>Shift left or right of the graph caused by adding or subtracting a value to the x term in a function

Example: $y = (x - 3)^3$
$y = (x + 4)^3$</td><td>Shift up or down of the graph caused by adding or subtracting a value to the entire function

Example: $y = (x - 3)^3 + 2$
$y = (x + 4)^3 - 3$</td></tr>
<tr><td>Horizontal Dilation f(k · x)</td><td>Vertical Dilation k · f(x)</td></tr>
<tr><td>Stretch or shrink of the graph caused by multiplying onto the x term in a function

Example: $y = (\frac{1}{2}x - 3)^3$
$y = (3x + 4)^3$</td><td>Stretch or shrink of the graph caused by multiplying onto the entire function

Example: $y = \frac{1}{2}(x - 3)^3$
$y = 3(x + 4)^3$</td></tr>
<tr><td colspan="2">Reflection – f(x)</td></tr>
<tr><td colspan="2">Flip of the graph caused by a negative being multiplied to the function

Example: $y = -2(3x + 4)^3$</td></tr>
</table>

Verbal Description → Equation
<u>Example 1:</u> What is the equation of the graph of y = x^3 under a vertical compression by a factor of ½ followed by a reflection over the x-axis, a horizontal translation of 3 units to the right and a vertical translation of 2 units up?

You Try It! Write the equation given the following description.

1.) What is the equation of the graph y = x^3 under vertical stretch by factor of 4, horizontal translation 2 units right and vertical translation 2 units down.

Equation → Verbal Description

Example 2: How has the function y = x^3 been transformed?

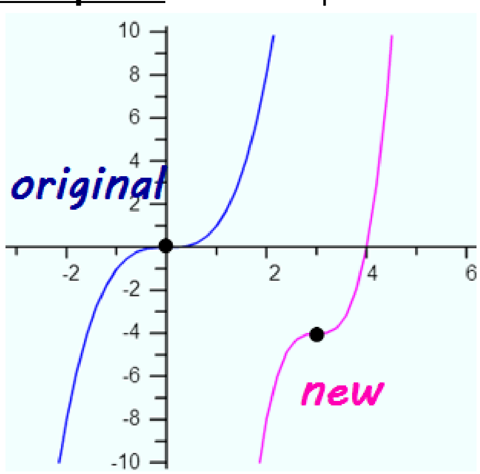
$$y = -4(x + 6)^3 - 5$$

You Try It! Describe the transformation of y = x^3

2.) $y = -2(x - 3)^3 + 4$

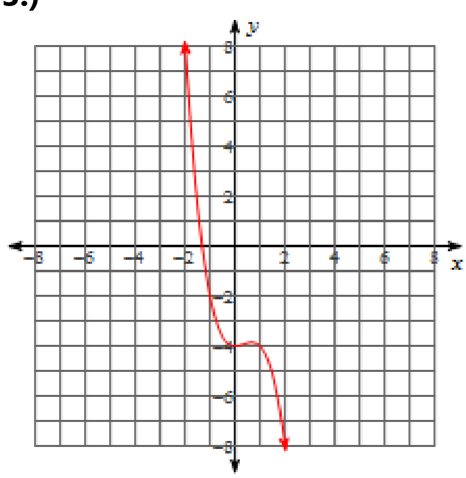
Graph → Equation

Example 3: Write an equation for the graph (translation/reflections only)



You Try It! Write the equation of the following graph.

3.)



#4	**Operations on Polynomials** **Text: 6.6** ☐ I can simplify polynomial expressions ☐ I can multiply polynomials ☐ Put functions together using addition, subtraction, multiplication, and division Vocabulary: box method, distributive property

Definitions	
LIKE TERMS are terms that share the same V_________________ with the same E__________________.	**Example 1:** Combine like terms $9 - x^2 + 4x + 2x^3 - x + 5$

Adding & Subtracting Polynomials *Just combine like terms*	
Example 2: $(2x^2 + x - 3) + (x - 1)$	**Example 3:** $(4x^3 + 2x^2 - 9) - (3x^2 + 4)$ *Note: When you subtract distribute the -1*

Multiplying Polynomials *Use the box method*	
A) Put factors on outside of box B) Multiply to fill box C) Combine like terms	**Example 4:** $(3x^2 + 2x + 4)(3x - 5)$

You Try It! Perform the indicated operations

1.) $(3x + 2x^4 - 5) + (2x - 1)$

2.) $(4x^3 - 3x + 9) - (2x^3 - 4)$

3.) $(6x^4 - 3x^2 + 7)(3x + 8)$

#5	**Solving Polynomials** **Text: 5.3** ☐ Explain why the x-coordinates of the points where the graphs of two functions meet are solutions Vocabulary: no new vocabulary

Solving Polynomials

Factor out GCF, then factor or use Quadratic Formula

For help with factoring/solving by factoring, see notes for Unit 2 Concept 3 & 4

For help with the Quadratic Formula, see notes for Unit 2 Concept 6

Example 1: What are the real or imaginary solutions of the equation?

$$15x^4 - 20x^3 - 35x^2 = 0$$

Example 2: What are the real or imaginary solutions of the equation?

$$6x^2 + 11x - 10 = 0$$

Example 3: What are the real or imaginary solutions of the equation?

$$2x^3 - 5x^2 = 4x$$

Special Cases
Example 4: $3x^3 - 27x = 0$
Example 5: $2x^4 - 20x^3 + 50x^2 = 0$

You try it! What are the real or imaginary solutions of the equation?

1.) $6x^2 + 13x - 5 = 0$

#6	**Dividing Polynomials** **Text: 5.4** ☐ Use the box method to divide a polynomial by (x-a). ☐ Write the answer in the correct form with any remainder written over the (x-a). ☐ Determine if (x-a) is a factor of a polynomial ☐ Divide a polynomial by a polynomial of the same or lesser degree ☐ Write the answer in the correct form with any remainder written over the bottom polynomial Vocabulary: dividend, divisor, quotient, remainder

Definitions	
DIVIDEND is the number you divide I__________ ***12*** ÷ 5 = 2 R2	**DIVISOR** is the number you divide B______ 12 ÷ ***5*** = 2 R2
QUOTIENT is the A___________ to a division problem 12 ÷ 5 = ***2*** R2	**REMAINDER** is what is L_________ after you divide 12 ÷ 5 = 2 ***R2***

Dividing Polynomials *Use the Box Method*	
A. Fill in any missing terms **B.** Put divisor on side of box **C.** Put 1^{st} term of dividend inside box **D.** "Reverse engineer" the rest! **E.** If last term inside box doesn't match dividend then have remainder	**Example 1:** $(4x^2 + 23x - 16) \div (x + 5)$

A. Fill in any missing terms **B.** Put divisor on side of box **C.** Put 1st term of dividend inside box **D.** "Reverse engineer" the rest! **E.** If last term inside box doesn't match dividend then have remainder	**Example 2:** $(x^6 - 3x + 5) \div (x - 1)$

You Try It! Use the box method to divide the polynomial

1.) Use polynomial division to divide $3x^2 – 29x + 56$ by $x – 7$. What is the quotient and remainder?

Determine whether (x – a) is a factor *Plug in a for x. If you get ZERO, x – a IS A FACTOR.*
Example 3: Is $(x - 1)$ a factor of $(3x^4 – 4x^3 + 12x^2 + 5)$?

You Try It!

2) Is $(x + 1)$ a factor of $(x^3 + 4x^2 + x – 6)$?

#7	**Using Polynomial Identities** **Text: Concept Byte 5.5** ☐ Show that the two sides of polynomial "formula" are equal ☐ Use the three polynomial "formulas" to explain numerical relationships Vocabulary: numerical relationship, identity

Polynomial Identities • Difference of Squares • Sum of Cubes • Differences of Cubes How to Find • Identify "a" and "b" • Look for pattern	**Difference of Squares** $a^2 - b^2 = (a + b)(a - b)$ **Example 1:** $x^2 - 81$
Sum of Cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ **Example 2:** $x^3 + 27$	**Differences of Cubes** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ **Example 3:** $x^3 - 216$

You Try It! Use the polynomial identities to factor each expression

1) $x^2 - 25$

2) $x^3 + 64$

3) $a^3 - 125$

#8

Polynomial Models in the Real World — Text: 5.8

- ☐ Explain properties of a real world situation from complex equations
- ☐ Relate the domain of a function to its graph
- ☐ Identify the input values that make sense in a real-world situation
- ☐ Calculate the average rate of change of a function from an equation and/or table
- ☐ Interpret and estimate the average rate of change of a function given a graph

Vocabulary: domain, rate of change, slope, interval

Example 1: Write a variable expression in both factored form and standard form for the area of a square whose side is x + 8

Example 2: You can represent volume with a cubic function. If you start with a rectangular piece of cardboard, cut out a square from each corner, and then fold up the sides, you will have an open top rectangular solid.

A. If you start with an 8.5 in. by 11 in. piece of cardboard, what is the length, width and height of the rectangular solid if you cut out a 1 in. by 1 in. square from each corner?

B. What is the volume of the box?

C. If you cut an x in. by x in. square from each corner, what is a general equation for the volume of the rectangular solid? Write this in both factored form and standard form?

Example 3: A section of a bridge can be modeled by the function $f(x) = x^2 - 21x + 98$. Support beams for this bridge will be placed at one of the zeros. What are the possible locations for the support beams?

Example 4: The dimensions of a window are $2x + 10$ and $x + 3$. What is the perimeter of the window?

Example 5: The expression $V(x) = x^3 - 13x + 12$ represents the volume of a rectangular safe in cubic feet. The length of the safe is $(x + 4)$. What linear expressions with integer coefficients could represent the other dimensions of the safe? Assume that the height is greater than the width.

Example 6: An arrow is shot upward. Its height *h*, in feet, is given by the equation $h = -16t^2 + 32t + 5$, where *t* is time in seconds. The arrow is released at t = 0s.
A. How many seconds does it take until the arrow hits the ground?
B. How high is the arrow after 2 seconds?