# Unit 1 Guided Notes

**Functions, Equations, and Graphs**

Standards: A.CED.2, A.CED.3, A.REI.11, A.SSE.1, F.BF.1, F.BF.3, F.IF.7, F.IF.8, F.IF.

Clio High School – Algebra 2A

- Miss Seitz’s tutoring: Thursdays after school
- Website with all videos and resources: [www.msseitz.weebly.com](http://www.msseitz.weebly.com)

### Need help? Support is available!

- Miss Kari Seitz
- **Text**: 810.309.9504
- **Classroom**: 810.591.1412
- **Email**: kseitz@clioschools.org

---

<table>
<thead>
<tr>
<th>Concept #</th>
<th>What we will be learning...</th>
<th>Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>Introduction to Functions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Compare properties of two functions each represented in different ways</td>
<td>2.1</td>
</tr>
<tr>
<td>#2</td>
<td>Linear Functions in Slope-Intercept Form</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Write linear equations in slope-intercept form</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>□ Draw a graph of an equation</td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td>More About Linear Functions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Manipulate an expression in order to reveal and explain different properties</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>□ Change the value of part of an expression and analyze how it changes the whole expression</td>
<td></td>
</tr>
<tr>
<td>#4</td>
<td>Graphing Linear Equations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Create appropriate axes with labels and scales with given information</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>□ Draw a graph of an equation</td>
<td>2.4</td>
</tr>
<tr>
<td>#5</td>
<td>Piecewise Functions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Graph piecewise functions</td>
<td>CB</td>
</tr>
<tr>
<td></td>
<td>□ Write equations of piecewise functions</td>
<td>2.4</td>
</tr>
<tr>
<td>#6</td>
<td>Absolute Value Functions and Step Functions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Graph absolute value and step functions</td>
<td>2.7</td>
</tr>
<tr>
<td>#7</td>
<td>Transformations of Graphs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative)</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>□ Find the value of $k$ given the graphs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Recognize even and odd functions from their graphs and algebraic expressions</td>
<td></td>
</tr>
<tr>
<td>#8</td>
<td>Analyzing Linear Models</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Interpret parts of an expression in real-world context</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>□ Write a function that describes a relationship between two quantities</td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>Linear Programming</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ Represent constraints by equations or inequalities, and by systems of inequalities/equations</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>□ Interpret solutions as viable or non-viable options in a modeling context</td>
<td></td>
</tr>
</tbody>
</table>
# Introduction to Functions

- **Text:** 2.1
- **Vocabulary:** function, domain, range, function notation

## Definitions

A **function** is a relation in which each element in the domain corresponds to exactly one element in the range. This is also called a **one to one** relationship.

- **Domain** is all possible **x**-values of a function.
- **Range** is all possible **y**-values of a function.

## Four Ways to Represent a Function

### 1.) Mapping Diagram

A mapping diagram **shows a function** if each element of the Domain maps to **one** element of the Range.

A mapping diagram **does NOT show a function** if ONE element of the Domain maps to **more than one** element of the Range.

### 2.) Ordered Pairs

Ordered pairs **show a function** if the Domain values do not repeat in the Range.

Ordered pairs **do NOT show a function** if the Domain values repeat in the Range.
Four Ways to Represent a Function

3.) Table of Values

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>-8</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>0</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

A table of values **shows a function** if the X-V_________ do NOT R_____________.

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>-8</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>0</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

A table of values **does NOT show a function** if the X-V_________ do R_____________.

4.) Graph

A graph **shows a function** if it passes the V_________ L_________ T_________.

A graph **does NOT show a function** if it does NOT pass the V_________ L_________ T_________.

Is Each a Function?

**Example 1:** Is each of the following a function?

A. ![Diagram A](image)

B. ![Diagram B](image)

C. ![Diagram C](image)

You Try It! Is each a function?

1.) \{ (1, 3), (2, -5), (3, -13) \}

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>4</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>5</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>
**Finding Domain and Range**

**Example 2:** What is the domain and range of the function?

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-3</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

**You Try It!** What is the domain and range of the function?

3.) \{(1, 3), (2, -5), (3, -13)\}

4.)

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>4</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>5</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

**Function Notation**

\[ f(x) = \underline{\text{__________}} \quad \text{It's just another way to write ______!} \]

**Example 3:** Given \( f(x) = -4x + 1 \), Find the value of \( f(-2) \)

To evaluate a given function at a particular value, P________ in the V________ for the V________ and do the C______________!

**You Try It!**

5.) Given \( f(x) = 3x - 5 \), Find the value of \( f(6) \)
### Linear Functions in Slope-Intercept Form

- Write linear equations in slope-intercept form
- Draw a graph of an equation

**Vocabulary:** linear function, slope, slope – intercept form, y-intercept

#### Definitions

A **linear function** is a special type of function whose graph is a straight line.

#### Slope

The slope of a non-vertical line through points \((x_1, y_1)\) and \((x_2, y_2)\) is the ratio of the vertical change to the corresponding horizontal change.

\[
\text{slope} = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_2 - x_1 \neq 0
\]

**s** is the ratio of the **v** change over the **h** change.

**Positive Slope**

- Line rises from left to right

**Negative Slope**

- Line falls from left to right

**Zero Slope**

- Horizontal line

**Undefined Slope**

- Vertical line

#### Example 1:

Use the graph at the right. Draw a line from the **slope** in Column A to the line with that slope in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive</td>
<td>line (a)</td>
</tr>
<tr>
<td>negative</td>
<td>line (b)</td>
</tr>
<tr>
<td>zero</td>
<td>line (c)</td>
</tr>
<tr>
<td>undefined</td>
<td>line (d)</td>
</tr>
</tbody>
</table>
Finding Slope Given Two Points

**Example 2:** Find the slope of the line between (-3, 7) and (-2, 4).

**You Try It!** Find the slope of the line with the given points.

1.) Line A from **Example 1** (hint: pick two points on the line)

2.) Between (2, 5) and (1, 8)

Slope-Intercept Form

The **Slope-Intercept Form** of an equation of a line is $y = mx + b$, where $m$ is the slope of the line and $(0, b)$ is the y-intercept.

**Example 3:** Graph $y = -2x + 1$

**Steps:**
1. Plot the y-intercept
2. Use the slope (rise/run)
3. Draw a line through the two points

**You Try It!** Graph the equation

3.) Graph $y = \frac{1}{2} x - 4$
More about Linear Functions

- Manipulate an expression in order to reveal and explain different properties
- Change the value of part of an expression and analyze how it changes the whole expression

Vocabulary: point-slope form, standard form, parallel, perpendicular

### Point-Slope Form

The equation of a line in **Point-Slope Form** through point \((x_1, y_1)\) with slope \(m\) is:

\[ y - y_1 = m(x - x_1) \]

**Derive Point-Slope Form:**

\[ m = \frac{y - y_1}{x - x_1} \]

**Example 1:** A line passes through \((-5, 2)\) and has slope \(3/4\). Write an equation for this line.

### Standard Form

The equation of a line in **Standard Form** is \(Ax + By = C\), where \(A\), \(B\), and \(C\) are real numbers, \(A\) is not negative, and \(A\) and \(B\) are not both zero.

**Example 2:** Write the equation of the line \(y = \frac{3}{4}x - 5\) in standard form.

### Writing Equations of Lines Summary

<table>
<thead>
<tr>
<th>Slope-Intercept Form</th>
<th>Point-Slope Form</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y = mx + b)</td>
<td>(y - y_1 = m(x - x_1))</td>
<td>(Ax + By = C)</td>
</tr>
<tr>
<td>Use this form when you know the (s)_______ and the (y)-________.</td>
<td>Use this form when you know the (s)_______ and a (p)_______ or when you know two (p)_______.</td>
<td>A, B &amp; C are real numbers A is positive A &amp; B cannot both be zero</td>
</tr>
</tbody>
</table>
**Example 3:** A line goes through (3, 1) and (4, 2). Find the equation of the line in **ALL THREE FORMS!**

**Point-Slope Form:**

**Standard Form:**

**Slope-Intercept Form:**

---

<table>
<thead>
<tr>
<th>Parallel Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel Lines have the same s__________, but different y-____________.</td>
</tr>
</tbody>
</table>

**Example 4:** Write the equation of the line parallel to the line 4x + 2y = 7 through (4, -2)

**Steps:**
1. Put the original equation in Slope-Intercept Form
2. Write the new equation in Point-Slope Form using $m$ from the original equation and the given point
3. Put in Slope-Intercept Form
Perpendicular Lines

Perpendicular Lines have one perpendicular r______ s_______.

**Example 5:** Write the equation of the line perpendicular to the line $y = \frac{2}{3}x - 1$ through (0, 6)

Steps:

<table>
<thead>
<tr>
<th>Old Slope:</th>
<th>New Slope:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old Slope:</td>
<td>New Slope:</td>
</tr>
</tbody>
</table>

1. Find the new slope

2. Write the new equation in Point-Slope Form using your new $m$ and the given point

3. Put in Slope-Intercept Form

**You Try It!** Write the equation of each in Slope-Intercept Form.

1. ) Parallel to $y = \frac{1}{3}x - 6$ through (-1, 6)

2. ) Perpendicular to $y = 2x + 5$ through (1, 4)
Graphing Linear Equations

- Create appropriate axes with labels and scales with given information
- Draw a graph of an equation

Vocabulary: intercepts

## Graphing a Line Using a Table

**Example 1:** Graph $y = -2x + 5$ using a table

**Steps:**
1. Draw the table
2. Choose 5 $x$-values
3. Plug $x$-values into the equation to get $y$-values
4. Plot and connect points on a graph

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-2x + 5$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## Graphing a Line Using Slope-Intercept Form

**Example 2:** Graph $y = \frac{1}{2}x + 3$

**Steps:**
1. Identify the slope and $y$-intercept
2. Plot the $y$-intercept on the graph
3. Use the slope (rise/run) to find the next point
4. Connect the points

## Graphing a Line Using Point-Slope Form

**Example 3:** Graph $y - 4 = 3(x + 2)$

**Steps:**
1. Identify the slope and point $(x_1, y_1)$
2. Plot $(x_1, y_1)$
3. Use the slope (rise/run) to find the next point
4. Connect the points

## Graphing a Line Using Standard Form (Using Intercepts)

**Example 4:** Graph $3x + 2y = 12$

1. Set $x = 0$ to find the $y$-intercept
2. Set $y = 0$ to find the $x$-intercept
3. Plot the intercepts
4. Connect the points
### Definitions
A **P**__________ **F**__________ is a function which is defined by sub-functions that each applies to a specific part of the domain. So the graph is broken into “pieces”. Hence the name!

### Graphing a Piecewise Function

#### **REMINDER***
- When you have < or >, you will have an **O**_____ **C**_____ at the point.
- When you have ≤ or ≥, you will have a **C**_____ **C**_____ at the point.

#### Example 1: Graph
\[
f(x) = \begin{cases} 
2x + 1 & \text{if } x < 0 \\
2x - 1 & \text{if } x \geq 0 
\end{cases}
\]

#### Steps:
1. Draw boundary lines at the “breaks”.
2. Graph the function for the first interval (2x + 1 if x < 0)
   - Open or closed circle?
3. Graph the function for the second interval (2x – 1 if x ≥ 0)
   - Open or closed circle?

#### You Try It!
Graph the following functions:

1. \[f(x) = \begin{cases} 
3x + 1 & \text{if } x < -1 \\
x - 3 & \text{if } x \geq -1 
\end{cases}\]
2. \[f(x) = \begin{cases} 
x + 1 & \text{if } x < 1 \\
-2x + 4 & \text{if } x \geq 1 
\end{cases}\]
### Writing a Piecewise Function

**Example 2:** Write the equation for the piecewise function below

**Steps:**

1. Find your intervals
   - 1\(^{\text{st}}\) interval:
   - 2\(^{\text{nd}}\) interval:
   - 3\(^{\text{rd}}\) interval:

2. Pick two points on each interval. Use them to find the slope of the line.

3. Use one of the points and the slope to write the equation of the line in Point-Slope Form.

4. Change to Slope-Intercept Form

<table>
<thead>
<tr>
<th>Work for 1(^{\text{st}}) Interval</th>
<th>Work for 2(^{\text{nd}}) Interval</th>
<th>Work for 3(^{\text{rd}}) Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Equation:**

**You Try It!** Write the equation of the piecewise function below
Definitions

Think of the absolute value as the d____________ f________ z_________
That’s why it is always p_____________!

Example 1: Use a table of values to help graph the function f(x) = -2|x|

| x   | -2|x| | y |
|-----|-----|---|
| -3  | 6   |   |
| -2  | 4   |   |
| -1  | 2   |   |
| 0   | 0   |   |
| 1   | 2   |   |
| 2   | 4   |   |
| 3   | 6   |   |

Domain: ___________________ Range: ______________

Even and Odd Functions

An E_______ F____________ is symmetric about the y – axis.

An O_______ F____________ is symmetric about the origin
(it looks the same if it’s flipped over the x-axis and then the y-axis)

Example 2: Even or odd?

Example 3: Even or odd?
**Step Functions**

A step function is a function whose graph looks like a bunch of steps. The most common step functions are the \( \text{F} \)_________ \( \text{F} \)_________ and the \( \text{C} \)_________ \( \text{F} \)_________.

<table>
<thead>
<tr>
<th>The <strong>Floor Function</strong> takes whatever number you put in for ( x ) and rounds it <strong>D</strong>_______ to the nearest <strong>integer</strong>.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Floor Function is written ( f(x) = \lfloor x \rfloor )</td>
</tr>
</tbody>
</table>

**Example 4:** What is the floor of each number?

| -1.1  |
| 0    |
| 1.01 |
| 2.9  |
| 3    |

<table>
<thead>
<tr>
<th>The <strong>Ceiling Function</strong> takes whatever number you put in for ( x ) and rounds it <strong>U</strong>____ to the nearest <strong>integer</strong>.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Ceiling Function is written ( f(x) = \lceil x \rceil )</td>
</tr>
</tbody>
</table>

**Example 5:** What is the ceiling of each number?

| -1.1  |
| 0    |
| 1.01 |
| 2.9  |
| 3    |

**You Try It!** Evaluate each

1.) \( \lfloor -2.0001 \rfloor \)
2.) \( \lceil 2.0001 \rceil \)
### Transformations of f(x)

#### Vertical Translations (shifts)

| Translation up \( k \) units | \( y = f(x) + k \) |
| Translation down \( k \) units | \( y = f(x) - k \) |

**Example:**
- \( f(x) = |x| + 4 \) shifts 4 units ______
- \( f(x) = x - 6 \) shifts 6 units ______

#### Horizontal Translations (shifts)

| Translation right \( h \) units | \( y = f(x - h) \) |
| Translation left \( h \) units | \( y = f(x + h) \) |

**Example:**
- \( f(x) = (x + 3) \) shifts 3 units to the ______
- \( f(x) = |x - 5| \) shifts 5 units to the ______

#### Vertical Stretches and Compressions/Shrinks

| Vertical Stretch, \( a > 1 \) | \( y = a \cdot f(x) \) |
| Vertical Compression (shrink), \( 0 < a < 1 \) | \( y = a \cdot f(x) \) |

**Example:**
- \( f(x) = 3x \) ______the graph by a factor of 3
- \( f(x) = \frac{1}{4}x \) ______or_______the graph by a factor of \( \frac{1}{4} \).

#### Reflections (flips)

| In the \( x \)-axis | \( y = -f(x) \) |
| In the \( y \)-axis | \( y = f(-x) \) |

**Example:**
- \( f(x) = -|x + 5| \)  
  Flip about the \___-axis occurs if the \E______ F_______ is made negative.
- \( f(x) = -|x| + 5 \)  
  Flip about the \___-axis occurs if \O_______ ___ is made negative.
Describing Transformations

**Example 1:** Describe how the parent function \( f(x) = |x| \) must be changed to graph the function \( y = 2|x - 1| + 3 \)

<table>
<thead>
<tr>
<th>What has changed?</th>
<th>So what happens to the graph?</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ 2 is being ______________</td>
<td>✓ ____________________</td>
</tr>
<tr>
<td>✓ -1 is being ___________</td>
<td>✓ ____________________</td>
</tr>
<tr>
<td>✓ 3 is being ____________</td>
<td>✓ ____________________</td>
</tr>
</tbody>
</table>

Identifying the Transformation Given the Graph

**Example 2:** Write the equation of the new function

Steps:
1. Identify what has changed
2. Write the equation

Parent Function: \( y = \sqrt{x} \)
(pink is new function)

**Example 3:** Write the equation of the new function

When it’s Stretched/Shrunk:

To find the value of the multiplier, we need to create and solve an equation using the parent function. Pick a point on the new graph and plug in the x and y coordinates to our new equation. We will use this to solve for our unknown, \( u \).

\[ y = u x^2 \]

Parent Function: \( y = x^2 \)
(pink is new function)
# Analyzing Linear Models

- Interpret parts of an expression in real-world context
- Write a function that describes the relationship between two quantities

Vocabulary: coefficient

## Definitions

A C__________ is the number in front of the variable.

### Example 1:
Name the coefficients of the following:

- \( y = 3x + 2 \)
  - Coefficient of \( X \): __
  - Coefficient of \( Y \): ___

- \( 4x - 2y = 10 \)
  - Coefficient of \( X \): ___
  - Coefficient of \( Y \): ___

- \( y = 4x - 2 \)
  - Coefficient of \( X \): ___
  - Coefficient of \( Y \): ___

## Writing Functions to Describe Relationships

### Example 2:
Write an equation for the situation. Phillip bought a roll of raffle tickets for $10. He will be selling 50-50 raffle tickets for $1 each. How much money, \( m \), will he make if he sells \( t \) tickets?

**Given:**

**Find:**

### Example 3:
The number of boxes, \( b \), in a warehouse is given by the equation \( b = 100d + 800 \) where \( d \) represents the number of days gone by. What do the coefficients in the equation represent?

- ✓ What does the 100 mean?
- ✓ What does the 800 mean?

## You Try It!
Write an equation for each situation

1.) Shelly wants to buy Legos. She is told the cost, \( c \), will be \( c = 7.35p + 5 \) where \( p \) represents the weight of her Lego purchase in pounds.
   - a. What does the number 7.35 represent?
   - b. What might the number 5 represent?

2.) Yahn is climbing a rope. His height, \( h \), above the ground is given by the equation \( h = 10t + 2 \) where \( t \) represents time measured in minutes and \( h \) is measured in feet.
   - a. What does the number 10 represent?
   - b. What does the number 2 represent?
Linear Programming

- Represent constraints by equations or inequalities, and by systems of equations/inequalities
- Interpret solutions as viable or nonviable options in a modeling context

Vocabulary: constraint, viable solution, nonviable solution

### Definitions

A **C**_______ is a factor which restricts a system

### Example 1: List all constraints.

For your rock collection display, you want to have at most 25 samples. You want to have at least three times as many sedimentary samples as metamorphic samples.

### Example 2: List all constraints.

An exam has two sections; a multiple choice section and an essay section. You can score a maximum of 100 points. You must get at least 65 points on the essay to pass the course.

### You Try It!

1.) Suppose you are buying two kinds of notebooks. A spiral notebook costs $2 and a 3-ring binder costs $5. You must have at least 6 notebooks. The cost of notebooks can be no more than $20.

### Checking for Viability

A **V**_______ **S**_______ is a solution which does not violate any constraints of a system

A **N**_______ **S**_______ is a solution which violates a constraint of a system

### Example 3: Given a list of constraints, tell whether a given solution is viable or not. If not, identify the constraint(s) which is/are not met

Constraints: -4x + 7y ≥ 21; 3x + 7y ≤ 28
Solution: (2, 3)
You Try It!  Given a list of constraints, tell whether a given solution is viable or not. If not, identify the constraint(s) which is/are not met

2.) Constraints: $-4x + 7y \geq 21$;  
$3x + 7y \leq 28$
Solution: (0, 4)

3.) Is the solution (3,1) viable with the following
Constraints: $x \leq 3$, $y \leq 5$, $x + y \geq 1$